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Ingredients and Equations for making a Magnetic Field in the Early Universe

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Abstract

The ingredients required to create a magnetic field in the early Universe are identified, and compared with Sakharov's conditions for baryogenesis. It is also shown that a long range coherent magnetic field is not generated by the classical rolling Higgs vacuum expectation value during the electroweak phase transition.

The Universe is observed to have coherent magnetic fields over a wide variety of scales, extending from the earth to possibly galaxy clusters [1]. These observations are puzzling, because it is not clear how the large-scale fields are made.

Our galactic disk locally has a coherent magnetic field of strength $\sim 10^{-6}$ Gauss, approximately in the direction of the galactic arm [1, 2]. Observations of the Faraday rotation of radio waves from extragalactic sources by various spiral galaxies suggest that these galaxies today have aximuthal magnetic fields of strength $\sim 3 \times 10^{-6}$ Gauss. There are also recent data suggesting micro Gauss fields are common in galaxy clusters. Measuring magnetic fields at high redshift is difficult, but it appears that there is a galaxy at $z \sim .4$ with a microGauss field, and that the Lyman α forest may also include clouds with fields of about this strength [1]. An extensive review of the observations and astrophysics of magnetic field preservation and amplification can be found in [1].

An aximuthal field is a natural configuration inside a differentially rotating spheroid, and suggests that the present magnetic field in spiral galaxies may have been amplified by the dynamo mechanism [1, 3]. However, the timescale during which the dynamo grows the field, and by how much the field is magnified, are unclear, so although one knows that a seed field is required on galactic scales, its magnitude is not well defined ($\vec{B} \sim 10^{-10} - 10^{-24}$ Gauss). There are various astrophysical [1] and cosmological [4, 5, 6, 7, 8] scenarios for making this seed field.

This paper addresses two distinct issues related to the generation of magnetic fields. In the first section, the symmetries, or properties, of the early

Universe that are broken by the presence of magnetic fields are identified, and whether they are required to *generate* a field is discussed in the light of various models. Interestingly enough, these “broken properties”, which are C, CP , thermal equilibrium and rotational invariance, are very similar to Sakharov’s required ingredients for baryogenesis.

The second issue addressed by this paper is whether the rolling higgs vev at the electroweak phase transition (EPT) produces a coherent, horizon-scale magnetic field. One of the popular cosmological mechanisms for generating the seed magnetic field is due to Vachaspati [5], who pointed out that there are Higgs field contributions to the electromagnetic $F^{\mu\nu}$ during the EPT. This is frequently taken to mean that horizon-scale magnetic fields of strength $\sim \alpha T^2$ are created at the phase transition. In the second section of this paper, it is shown that the rolling Higgs vev is electrically neutral. This means that classically, there are no sources for the magnetic field during the phase transition, and no long range field is generated.

1 making a magnetic field

Three ingredients are necessary to make a baryon asymmetry: one must have baryon number violation, C and CP violation, and out of equilibrium dynamics. The familiar explanation for this is that baryon number is odd under C and CP , and there are no asymmetries of non-conserved quantities in a thermal bath. Reasoning by analogy, one would like to know what ingredients are required to make a magnetic field in the early Universe?

Let us first address a simpler question: not what is required to create a

magnetic field, but rather what symmetries are broken, or what conditions do not apply, in the presence of a magnetic field? This gives three conditions: one needs some kind of out-of-thermal-equilibrium dynamics, because in equilibrium the photon distribution is thermal, and there are no particle currents to sustain a “long range” field. One needs C and CP violation, because the \vec{B} field is odd under both, and the initial state of the Universe is expected to be even under these symmetries. And finally, one needs to break the isotropy of the early Universe, because the magnetic field chooses a direction. So the properties of the early Universe that are broken by the presence of a magnetic field (C , CP , $SO(3)$ and thermal equilibrium) are very similar to those broken by the baryon asymmetry [9].

The interesting question, however, is not what properties are broken by the presence of the magnetic field, but rather what ingredients are required to *make* it. In the case of the baryon asymmetry, the symmetries that are broken by the presence of the particle excess have to be broken to generate it. But for the magnetic field, this is not the case, because it is a classical field. It can “spontaneously” develop an expectation value, if it has an effective potential of the right shape. So although many models for the generation of a primordial seed magnetic field do contain explicit C , CP and $SO(3)$ breaking, this is in principle not *required*. The analogy between generating the baryon asymmetry and a magnetic field can clarify this. In baryogenesis, the desired end result is a C , CP and baryon number violating excess of particles over anti-particles. It is possible to generate this asymmetry using spontaneous CP [10] and baryon number [11] violation; in other words, the Lagrangian need not break either of these symmetries explicitly. However,

prior to the generation of the perturbative particle asymmetry, a classical field develops a baryon number and/or CP violating vev. So that from the point of view of making the excess of particles over anti-particles, baryon number and CP violation are required. The magnetic field, on the other hand, is a classical object, and can itself spontaneously break C , CP and $SO(3)$ as it develops. (Note that this implies, in principle, that one could use a magnetic field to provide the CP violation required in baryogenesis models.) The only requirement for magnetic field generation is therefore some out of thermal equilibrium physics.

Let us now consider various models for the generation of a \vec{B} field, to illustrate the above discussion. One way to generate a primordial magnetic field is to make an electric current, which has an associated magnetic field [6]. In this case, one does need explicit C , CP and $SO(3)$ violation, as well as some kind of out of equilibrium dynamics, because these are required to generate an electromagnetic current in the particles present in the hot early Universe. This can make it difficult to generate a sufficiently large seed field, because CP violation and out of equilibrium physics are hard to come by in the early Universe. Astrophysical mechanisms [1] for the generation of the seed magnetic field also make the field “via Maxwell’s equations”, and require that all these conditions be broken explicitly. However, this is not a significant constraint by the time galaxies are formed, because C , CP and $SO(3)$ are no longer approximately symmetries of the Universe. A primordial magnetic field can also be generated by amplifying fluctuations during the inflationary epoch [7]. The intuitive picture is that a small fluctuation in the electromagnetic field is inflated into a classical field. This is clearly a non-

equilibrium process, but no explicit symmetry breaking is required. Another mechanism that does not require explicit C , CP and $SO(3)$ violation is to amplify magnetic field fluctuations using the dynamo mechanism in the turbulent fluid present after a phase transition [4, 8].

In this section, it was argued that although magnetic fields are non-equilibrium configurations that break C , CP and $SO(3)$, the only ingredient required to generate them is some out-of-thermal-equilibrium process. This is in contrast to the baryon asymmetry, where the symmetries that are broken by the presence of the asymmetry have to be broken to generate it.

2 Maxwell's equations in the presence of a varying Higgs vev

During the electroweak phase transition, the Higgs vev is in the process of going from an $SU(2) \times U(1)$ symmetric ground state to the vacuum state we live in today. Vachaspati [5] has argued that it creates magnetic fields in this process. Electromagnetism is difficult to understand during the phase transition because the Higgs field, whose vev usually defines the direction of the unbroken symmetry, is space-time and gauge dependent. To understand the implications of the space-time variations in the Higgs, one must separate them from the gauge dependence.

't Hooft has given a gauge-invariant definition of the electromagnetic field $F_{\mu\nu}$ in the Georgi-Glashow model (where $SO(3)$ is broken to $U(1)$), in the presence of a non-trivial Higgs vev. Vachaspati [5] similarly defined a gauge-

invariant electromagnetic tensor for the Standard Model

$$F_{\mu\nu}^{em} = \sin \theta_W \eta^a W_{\mu\nu}^a + \cos \theta_W B_{\mu\nu} - i \frac{4 \sin \theta_W}{g \phi^\dagger \phi} [(\mathcal{D}_\mu \phi)^\dagger \mathcal{D}_\nu \phi - (\mathcal{D}_\nu \phi)^\dagger \mathcal{D}_\mu \phi] \quad (1)$$

where

$$\mathcal{D}^\mu = \partial^\mu - ig' \frac{Y}{2} B^\mu - ig \vec{T} \cdot \vec{W}^\mu \quad (2)$$

($T_i = \sigma_i/2$). He then argues that (1) is naturally non-zero during the phase transition, because the varying Higgs vevs are uncorrelated across the horizon. It is unclear (to this author) whether such a complicated expression is zero or not, although it does have simpler, but less obviously gauge invariant formulations. The purpose of writing this expression was to have a gauge invariant formulation, so arguing that the $B_{\mu\nu}$ and $W_{\mu\nu}$ part of this is zero, and the scalar part is not is a gauge dependent statement. It would be clearer to retain the explicit gauge independence. Let us therefore assume that Maxwell's equations are correct, so that one cannot generate a field without a source. In a purely classical analysis, the electromagnetic field evolves as

$$\partial_\mu F^{\mu\nu} = j^\nu \quad (3)$$

where j^ν is the current due to the rolling classical Higgs vev (and not the expectation value of the quantum operator j^ν). Therefore if $F_{\mu\nu} = 0$ at the beginning of the phase transition, it is zero at the end unless there is some source during the transition.

The point is now to show that one can write an $SU(2) \times U(1)$ gauge invariant definition of j^ν that is clearly zero during the phase transition.

In ordinary QED, for a singly charged scalar, the RHS of equation (3) is

$$j^\nu = ie\phi^* Q \mathcal{D}^\nu \phi - ie(\mathcal{D}^\nu \phi)^* Q \phi \quad (4)$$

where $\mathcal{D}^\nu = \partial^\nu - ieQA^\nu$, and Q is the charge operator. This can be generalized to $SU(2) \times U(1)$, following 't Hooft; if one varies the Standard Model Higgs kinetic term with respect to the photon field A^ν , one gets

$$\frac{\delta}{\delta A^\nu} (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) = ie\phi^\dagger Q \mathcal{D}^\nu \phi - ie(\mathcal{D}^\nu \phi)^\dagger Q \phi \quad (5)$$

where \mathcal{D}_μ is defined in (2), and Q is the gauge invariant generator in the direction of the unbroken symmetry, which is defined in equation (7). Under an $SU(2) \times U(1)$ gauge transformation U , one can see that

$$j^\nu \longrightarrow ie\phi^\dagger U^\dagger Q U \mathcal{D}^\nu \phi - ie(\mathcal{D}^\nu \phi)^\dagger U^\dagger Q U \phi \quad (6)$$

so j^ν is gauge invariant if $Q = U^\dagger Q U$. One can check that

$$Q = -2 \frac{\phi^\dagger T^a \phi}{\phi^\dagger \phi} T^a - \frac{Y}{2} \quad (7)$$

is gauge invariant under both hypercharge and $SU(2)$ transformations, and reduces to the usual definition of Q in the unitary gauge.

It is easy to check that the operator Q from equation (7) is the generator of the unbroken symmetry, ie

$$Q\phi = 0 \quad (8)$$

so that there is no electromagnetic current, and therefore no magnetic field, generated by the rolling Higgs vev during the EPT.

A possible caveat to this argument is that it only applies when $Q \neq 0$. However, the Standard Model has no topological defects, so one can imagine

deforming the Higgs slightly away from zero, and then this argument should be applicable. There are non-topological defects and other classical field configurations that have associated magnetic fields, but the coherence length is presumably much shorter than the horizon size, so that the random walk average on galaxy scales [4] produces too small a field today.

conclusion

It has been argued that the classical rolling Higgs at the electroweak phase transition does not generate a horizon-scale magnetic field. However, it is not excluded that thermal or quantum fluctuations about this time dependent vev could produce the \vec{B} field.

The symmetries and conditions broken by the presence of a magnetic field, namely thermal equilibrium, CP , C , and rotational invariance have been identified. It is possible for the magnetic field to spontaneously break C , CP and $SO(3)$ as it develops, so these symmetries do not need to be broken to generate \vec{B} . However, some out-of-thermal-equilibrium physics is required.

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